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to  
Astronomical Interferometry

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# **The Application of Fourier Transform Heterodyne to Astronomical Interferometry**

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## **Abstract**

New, general techniques exist for directly measuring the phase and magnitude of electromagnetic fields. These direct field imaging (DFI) technologies facilitate the measurement of the amplitude and phase of electromagnetic fields in wavelength regimes shorter than radio wavelengths. In principle, DFI can be used over the entire electromagnetic spectrum because the concept is inherent in the nature of electromagnetic fields. In practice, an experiment demonstrating imaging of the amplitude and phase of an electromagnetic wave at HeNe laser wavelengths (633 nanometers) has been carried out.

The ability to measure (image) the two components of the electric field opens up the possibility of accomplishing ground-based or space-based astronomical interferometry (AI) in the visible and infrared regime in direct analogy with very long baseline interferometry (VLBI), which is routinely carried out at radio wavelengths. In such a scenario, the magnitude and phase of the image field is coherently measured at each telescope in the array. Later the data from each telescope is brought together by coherently combining the properly phase matched signal data. The resulting image possesses resolution characteristic of the widest baselines between elements of the array: that is a property of synthesis imaging. As in current AI techniques, DFI requires detailed *knowledge* of the baseline distance between the separate telescopes. However, unlike current AI techniques, DFI does *not* require that the baseline distance be *maintained* to a fraction of a wavelength as long as the *knowledge* of the baseline distance is recorded.

## Fourier Transform Heterodyne

Electromagnetic field imaging requires a detection process in which both spatial amplitude and phase information is preserved. Eyes, film, CCD cameras and other photosensitive elements image light in a process referred to as square-law detection (Goodman). Square-law detection implies that the observer images the *intensity* of the field, with all phase information lost. Preservation of both spatial amplitude and phase suggests a detection process that sidesteps the limitations imposed by the square-law process.

A more complete and rigorous description of Fourier Transform Heterodyne (FTH) is given in Cooke, et al. FTH is based on Poynting's relation (Jackson) (Cohen) (Fowler et al.) that describes the current (I) induced in a detector to the electromagnetic fields impinging on it:

$$I = k \iint_s \eta (\mathbf{E} \times \mathbf{H}) \cdot d\mathbf{s}$$

where  $\mathbf{E}$  and  $\mathbf{H}$  are the vector electric and magnetic fields, respectively,  $\eta$  is the detector's quantum efficiency,  $s$  is the detector's surface area and  $k$  is a constant.

The FTH technique involves creating a set of reference fields, in which the phase distributions constitute an orthonormal basis set. This set of functions is then used to sequentially interrogate an unknown signal field of slightly different wavelength. That is, each function in the basis set is heterodyned, one at a time, with the signal wave front. The resulting measurement of each heterodyne signal in a single element detector is the intermediate frequency's (IF) amplitude and phase shift. Analytically, for the set of conditions:

- TEM fields which means  $E_z=0$ ,  $H_z=0$  and  $\mathbf{H}=\sqrt{(\epsilon/\mu)}*\mathbf{E}$
- $A$ ,  $k$  and  $\eta$  constants
- $\mathbf{E} = \mathbf{E}_{\text{signal}} + \mathbf{E}_{\text{ref}}$  and  $\mathbf{H} = \mathbf{H}_{\text{signal}} + \mathbf{H}_{\text{ref}}$
- $E_{\text{signal}} = \rho(x,y) \exp[-i\omega t - i\phi(x,y)]$ ,  $E_{\text{ref}} = A \exp[i\omega' t - i\theta(x,y)]$
- $ds = dx dy$

the IF current part of Poynting's relation becomes:

$$I(k_x, k_y, \Delta\omega) = 2kA\eta \sqrt{\frac{\epsilon}{\mu}} \operatorname{Re} \left\{ \iint_s \rho(x,y) \exp[-i(\omega - \omega')t + i(\phi(k_x, k_y, x, y) - \theta(k_x, k_y, x, y))] dx dy \right\}$$

and simplifies to

$$I(k_x, k_y, \Delta\omega) = 2kA\eta \sqrt{\frac{\epsilon}{\mu}} \beta(k_x, k_y) \cos(\Delta\omega t + \alpha(k_x, k_y))$$

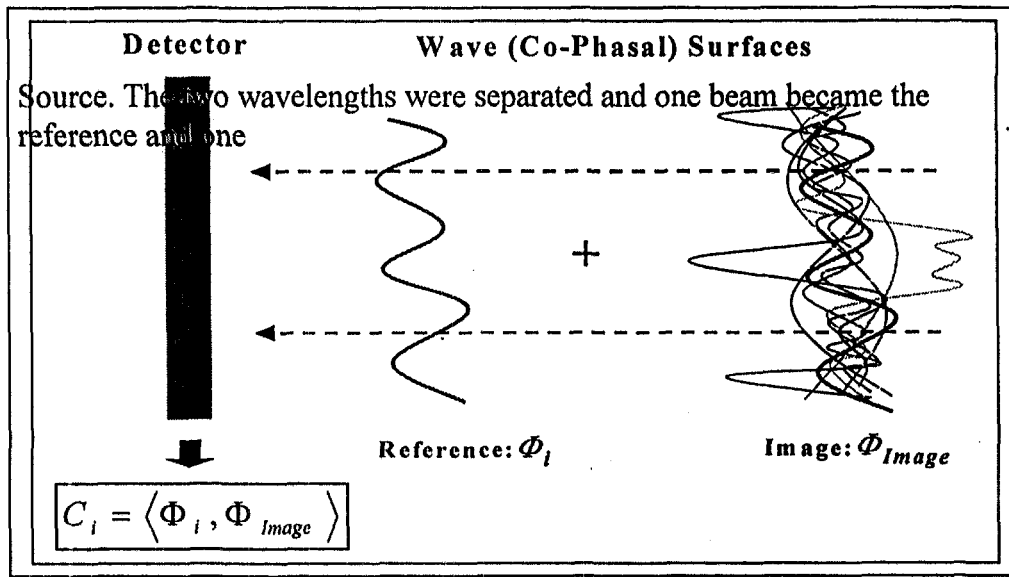
where  $\Delta\omega = \omega - \omega'$ ,  $\beta(k_x, k_y)$  is the amplitude of the IF current and  $\alpha(k_x, k_y)$  is the phase of the IF current (the integrated difference of  $\phi$  and  $\theta$ ) expressed as functions of the spatial frequencies,  $k_x$  and  $k_y$ . Conceptually, this process is a

dot product of the signal and each reference field. More rigorously, a series of Fourier projection operations have been performed (see figure 1). The amplitude and phase measurement of each IF is a complex coefficient,  $C(k_x, k_y)$ :

$$C(k_x, k_y) = \beta(k_x, k_y) e^{i\alpha(k_x, k_y)}.$$

The image reconstruction is accomplished by summing each of the coefficients times its basis field. Formally this is:

$$E_{imgrec}(x, y) = \sum_m \sum_n C_{m,n}(k_x, k_y) E_{ref,m,n}(x, y).$$



**Figure 1.** A conceptual view of phase projection. A complicated image field is combined with one term of a complete basis set of reference fields. This operation picks out the component in the image field that matches the reference field. The measured complex coefficient quantifies the admixture of this reference field component in the image field through its magnitude and phase.

The FTH proof-of-principle experiment was very simple (see figure 2). A helium-neon Zeeman split laser with a 250 kHz frequency shift between the two laser outputs constituted the signal. The reference beam was sent through a spatial light modulator upon which a set of thirty-six Zernike (Wang and Silva) functions was sequentially impressed. The signal beam passed through a target to introduce “structure” on the beam. The two beams were recombined and fell onto a single element silicon detector. The amplitude and phase of the IF signal were read by a computer interfaced to the oscilloscope (see figure 3).

The coefficients gathered on each target were used to reproduce the wave front’s amplitude and phase and hence an image of the target was obtained. The results were definitive. The critical target was a transmissive

microscope slide. The FTH phase reconstruction showed a  $0.9\pi$  phase shift in the proper location of the target plane thus proving that the phase had been retrieved. Based upon these results the team began to develop possible applications. The rest of this paper discusses one of these applications.

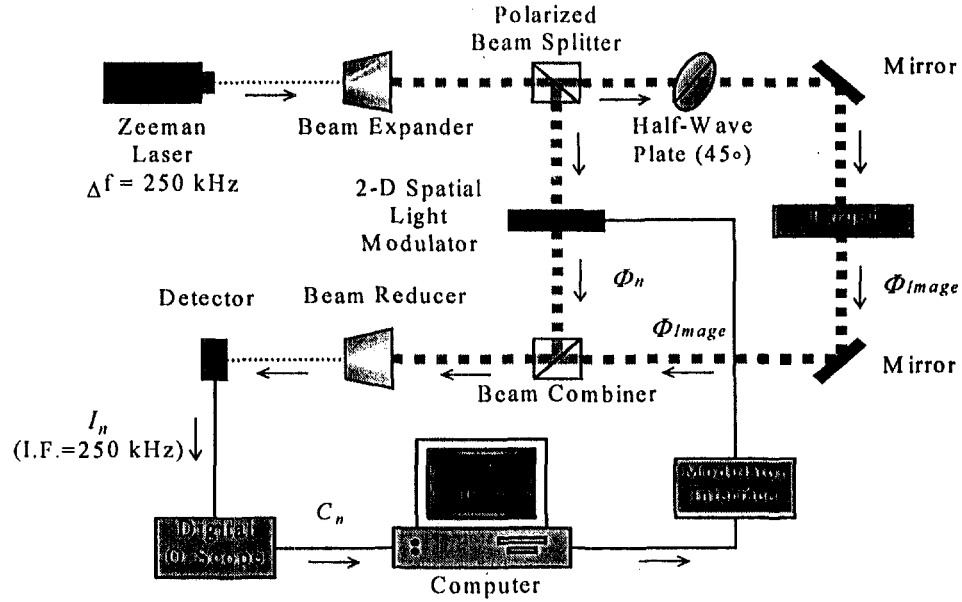


Figure 2. Schematic of the FTH experiment.

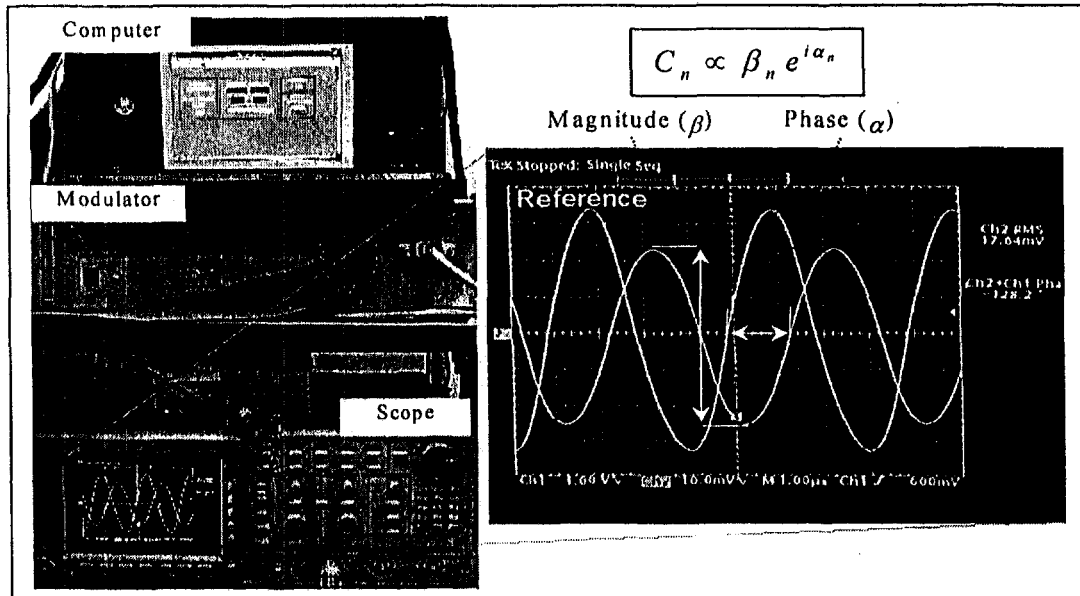


Figure 3. A digital oscilloscope measures the 250 kHz I.F. current's phase and magnitude from which Fourier coefficients,  $C_n$ , are formed.

## Astronomical Interferometry Issues

This paper concerns the application of DFI to astronomical interferometry (AI). The promise of DFI is that visible and infrared interferometry can be carried out in direct analogy with VLBI techniques of radio astronomy. That is, the amplitude and phase is recorded at each individual telescope of the array and later the images are calculated from the data using a computer. The advantage gained is that the baseline distances need not be maintained throughout the observation time – only the knowledge of the baseline distances during the short, discrete measurements.

Indeed, the greatest gains may be realizable in space-based interferometers. The scenario in this case involves free flying telescopes that are immune to the problems of truss vibrations. A laser metrology system continuously records the positions of the satellites but control of position (station keeping) requirements are relaxed since knowledge is required, rather than control. Timing considerations to ensure proper realignment of the data (so that the analysis combines data from the “same phase front”) remain. Possible solutions to this issue will be discussed below.

In this section the issues that arise when DFI is applied to AI will be described. These issues include:

- The longitudinal coherence length of starlight is very short because of the thermal nature of the star’s emission surface. This places constraints on the use of the DFI detection process since it is a coherent technique. The individual telescopes’ data must be properly “phase aligned” to within a fraction of this distance to interfere. These requirements constrain the position knowledge and timing. For 10 micrometer radiation of 1 micrometer bandwidth the corresponding longitudinal coherence length ( $l_L$ ) is:

$$l_L = \frac{\lambda^2}{\Delta\lambda} = 0.0001 \text{ meters}$$

- Alternatively, one can look at the longitudinal coherence time and thereby gauge the phasing constraints. The corresponding coherence time ( $t_L$ ) is:

$$t_L = \frac{l_L}{c} = 3.3 \times 10^{-13} \text{ seconds}$$

Optical Bandwidth ( $\Delta\lambda$ in microns) ( $\lambda=10.6$ microns)	Longitudinal Coherence Length (meters)	Longitudinal Coherence Time (sec)
10	0.00001	$3.3 \times 10^{-14}$
1	0.0001	$3.3 \times 10^{-13}$
0.1	0.001	$3.3 \times 10^{-12}$
0.01	0.01	$3.3 \times 10^{-11}$

- The transverse coherence length ( $l_r$ ) of the wave front from a star is large. For example, a wave front from Betelgeuse possessing a wavelength of 10 microns produces at the earth:

$$l_r = \frac{0.61\lambda}{\alpha} = \frac{(0.61)(10\mu m)}{0.047 \text{ arc sec}} = 26.7 \text{ meters}$$

where  $\alpha$  is the angle subtended by the disk of Betelgeuse as seen from Earth

- Maximization of spectral range used in the heterodyne is important to use as much of the starlight as possible. On the other hand, the greater the bandwidth, the greater the detection noise. The spectral range, centered about 10 micrometers, is related to the bandwidth of the detector. The table below illustrates the relationship - note that 100 GHz is the current state-of-the-art.

Bandwidth ( $\Delta\nu$ in GHz)	Spectral Range ( $\Delta\lambda$ in $\mu m$ )
100	0.033
1000	0.32
10,000	2.5

- Since DFI techniques are in their infancy (first publication 1998), supporting technologies such as spatial light modulators may not be readily available for some wavelength regimes. These technology bottlenecks need to be identified.
- The traditional techniques of interferometry must be compared and contrasted to the proposed DFI approach. Some of the advantages of DFI are the near-quantum limited performance of a heterodyne system as well as background rejection capability. Although DFI simplifies certain mission requirements, it may complicate other instrumentation requirements. A trade study is required to quantitatively sort out the pros and cons.

### Terrestrial Planet Finder Scenario

The extreme mission requirements and timeline of the NASA Terrestrial Planet Finder (TPF) mission dovetail with the capabilities and development time of the FTH technology. In the brief scenario laid out in this paper, the TPF mission is similar to some existing plans and has the following characteristics:

- The orbit is away from the earth at around 1 AU
- The individual telescope primary mirrors are around 4 meters in diameter
- Baselines of around 150 meters or greater may be required
- The TPF telescopes are each on free-flying spacecraft
- The telescopes are passively cooled to 35K

- The receiver systems operate around the 10 micrometer wavelength regime

Our proposed scenario is as follows. The receiver spacecraft (RS) number about ten and are arrayed in a roughly spherical pattern to maximize U-V plane coverage. The plane of this constellation is perpendicular to the target star.

Each telescope has a single element infrared sensor as the receiver system. This receiver system is an FTH receiver and as such performs a series of near-quantum limited heterodyne measurements on the stellar wavefront. The amplitude and phase of this wavefront is measured at each satellite and stored with a “time tag” so that when combined with the position knowledge, the data from each instrument can be recombined to produce the images. This operation is in direct analogy with the methods currently used in very long baseline interferometry (VLBI) radio frequency techniques.

A laser metrology system maintains and records in time the separations of the individual spacecraft in the plane that the constellation defines. Because the transverse coherence of the wavefront is very large, the constraints on the station keeping in this plane are much relaxed from the requirements in the longitudinal direction of the wavefront.

The key innovation to exploiting FTH in this scenario and resolving the longitudinal timing requirements is to use a master timing satellite (MTS). This satellite is stationed perpendicular to the constellation plane in the direction of the target many kilometers distant. Its functions in one of two ways:

- It generates a timing signal that rides along with the stellar wavefront and triggers each RS in the constellation to perform a measurement when the timing signal reaches the RS. In this way each RS measures the same wavefront (from the target star) as every other RS without regard to its distance from the idealized plane of the constellation.
- In a more involved scenario, the MTS generates the FTH reference oscillator pulse itself as the timing pulse. Then the RS telescopes perform the heterodyne automatically when the master oscillator pulse arrives at the receiver system. In the same way as in the previous way, the longitudinal timing is resolved by each RS measuring the same wavefront from the star as the other members of the constellation.

## Conclusion

This brief paper has outlined Direct Field Imaging theory, i.e., a new general technique for measuring the magnitude and phase of an electromagnetic field. Then in a cursory way its usefulness in space-based astronomical interferometry missions was proposed. The main advantage that these techniques bring to the complex problem of carrying out interferometry



from free-flying space-based telescopes is that the station keeping requirements are relaxed. This is because accurate knowledge of the positions of the telescopes is required during the short integration times. This is in contrast to traditional techniques wherein the array of telescopes must maintain a configuration (to an accuracy of a fraction of the coherence length of the radiation) throughout the entire observation period. With these techniques, interferometry can be carried out at visible and infrared wavelengths in direct analogy to radio interferometry.

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